7-4 *n*th Roots

Main Ideas

- Simplify radicals.
- Use a calculator to approximate radicals.

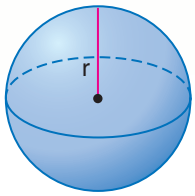
New Vocabulary

*n*th root

principal root

GET READY for the Lesson

The radius r of a sphere with volume V can be found using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$. This is an example of an equation that contains an nth root. In this case, $n = 3$.



Simplify Radicals Finding the square root of a number and squaring a number are inverse operations. To find the square root of a number n, you must find a number whose square is n.

Study Tip

Look Back

Review **square roots** in Lesson 5-4.

Similarly, the inverse of raising a number to the nth power is finding the **nth root** of a number. The table below shows the relationship between raising a number to a power and taking that root of a number.

Powers	Factors	Roots
$a^3 = 125$	$5 \cdot 5 \cdot 5 = 125$	5 is a cube root of 125.
$a^4 = 81$	$3 \cdot 3 \cdot 3 \cdot 3 = 81$	3 is a fourth root of 81.
$a^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	2 is a fifth root of 32.
$a^n = b$	$\underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a} = b$	a is an nth root of b.

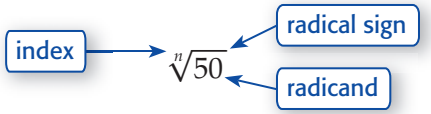
This pattern suggests the following formal definition of an nth root.

KEY CONCEPT — ***Definition of nth Root***

Word For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.

Example Since $2^5 = 32$, 2 is a fifth root of 32.

The symbol $\sqrt[n]{\ }$ indicates an nth root.



Some numbers have more than one real nth root. For example, 36 has two square roots, 6 and -6. When there is more than one real root, the nonnegative root is called the **principal root**. When no index is given, as in $\sqrt{36}$, the radical sign indicates the principal square root. The symbol $\sqrt[n]{b}$ stands for the principal nth root of b. If n is odd and b is negative, there will be no nonnegative root. In this case, the principal root is negative.

$\sqrt{16} = 4$ $\sqrt{16}$ indicates the principal square root of 16.

$-\sqrt{16} = -4$ $-\sqrt{16}$ indicates the opposite of the principal square root of 16.

$\pm\sqrt{16} = \pm 4$ $\pm\sqrt{16}$ indicates both square roots of 16. $\pm$ means positive or negative.

$\sqrt[3]{-125} = -5$ $\sqrt[3]{-125}$ indicates the principal cube root of -125.

$-\sqrt[4]{81} = -3$ $-\sqrt[4]{81}$ indicates the opposite of the principal fourth root of 81.

CONCEPT SUMMARY *Real nth roots of b, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$*

n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$b = 0$
even	one positive root, one negative root $\pm\sqrt[4]{625} = \pm 5$	no real roots $\sqrt{-4}$ not a real number	one real root, 0 $\sqrt[n]{0} = 0$
odd	one positive root, no negative roots $\sqrt[3]{8} = 2$	no positive roots, one negative root $\sqrt[5]{-32} = -2$	

Study Tip

Fractional Exponents

For any real number b and any positive integer n, $\sqrt[n]{b} = b^{\frac{1}{n}}$.

EXAMPLE Find Roots

1 Simplify.

a. $\pm\sqrt{25x^4}$

$\pm 25x^4 = \pm\sqrt{(5x^2)^2}$

$= \pm 5x^2$

The square roots of $25x^4$ are $\pm 5x^2$.

b. $-\sqrt{(y^2+2)^8}$

$-\sqrt{(y^2+2)^8} = -\sqrt{[(y^2+2)^4]^2}$

$= -(y^2+2)^4$

The opposite of the principal square root of $(y^2+2)^8$ is $-(y^2+2)^4$.

c. $\sqrt[5]{32x^{15}y^{20}}$

$\sqrt[5]{32x^{15}y^{20}} = \sqrt[5]{(2x^3y^4)^5}$

$= 2x^3y^4$

The principal fifth root of $32x^{15}y^{20}$ is $2x^3y^4$.

d. $\sqrt{-9}$

$\sqrt{-9} = \sqrt[2]{-9}$ (n is even. b is negative.)

Thus, $\sqrt{-9}$ is not a real number.

CHECK Your Progress

1A. $\pm\sqrt{81y^6}$ **1B.** $-\sqrt{(x-3)^{12}}$

1C. $\sqrt[6]{729x^{30}y^{18}}$ **1D.** $\sqrt{-25}$

When you find the nth root of an even power and the result is an odd power, you must take the absolute value of the result to ensure that the answer is nonnegative.

$$\sqrt{(-5)^2} = |-5| \text{ or } 5 \qquad \sqrt{(-2)^6} = |(-2)^3| \text{ or } 8$$

If the result is an even power or you find the nth root of an odd power, there is no need to take the absolute value. *Why?*

EXAMPLE Simplify Using Absolute Value

2 **Simplify.**

a. $\sqrt[8]{x^8}$

Note that x is an eighth root of x^8. The index is even, so the principal root is nonnegative. Since x could be negative, you must take the absolute value of x to identify the principal root.

$$\sqrt[8]{x^8} = |x|$$

b. $\sqrt[4]{81(a+1)^{12}}$

$$\sqrt[4]{81(a+1)^{12}} = \sqrt[4]{[3(a+1)^3]^4}$$

Since the index 4 is even and the exponent 3 is odd, you must use an absolute value.

$$\sqrt[4]{81(a+1)^{12}} = 3|(a+1)^3|$$

CHECK Your Progress

2A. $\sqrt{100x^{10}}$ **2B.** $\sqrt{64(y+1)^{14}}$

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Approximate Radicals with a Calculator Recall that real numbers that cannot be expressed as terminating or repeating decimals are *irrational numbers*. Approximations for irrational numbers are often used in real-world problems.

EXAMPLE

3 **PHYSICS** **The distance a planet is from the Sun is a function of the length of its year. The formula is $d = \sqrt[3]{6t^2}$, where d is the distance of the planet from the Sun in millions of miles and t is the number of Earth-days in the planet's year. If the length of a year on Mars is 687 Earth-days, how far from the Sun is Mars?**

$d = \sqrt[3]{6t^2}$ Original formula

$= \sqrt[3]{6(687)^2}$ or about 141.48 $t = 687$

Mars is approximately 141.48 million miles from the Sun.

CHECK According to NASA, Mars is approximately 142 million miles from the Sun. So, 141.48 million miles is reasonable. ✓

Study Tip

Graphing Calculators

To find a root of index greater than 2, first type the index. Then select $\sqrt[x]{\ }$ from the MATH menu. Finally, enter the radicand.

CHECK Your Progress

3. Approximately how far away from the Sun is Earth?

CHECK Your Understanding

Examples 1, 2 (pp. 403–404)

Simplify.

1. $\sqrt[3]{64}$
2. $\sqrt{(-2)^2}$
3. $\sqrt[5]{-243}$
4. $\sqrt[4]{-4096}$
5. $\sqrt[3]{x^3}$
6. $\sqrt[4]{y^4}$
7. $\sqrt{36a^2b^4}$
8. $\sqrt{(4x + 3y)^2}$

Example 3 (p. 404)

Use a calculator to approximate each value to three decimal places.

9. $\sqrt{77}$
10. $-\sqrt[3]{19}$
11. $\sqrt[4]{48}$

12. **SHIPPING** Golden State Manufacturing wants to increase the size of the boxes it uses to ship its products. The new volume N is equal to the old volume V times the scale factor F cubed, or $N = V \cdot F^3$. What is the scale factor if the old volume was 8 cubic feet and the new volume is 216 cubic feet?

Exercises

HOMEWORK HELP

For Exercises	See Examples
13–22	1
23–36	2
37–50	3

Simplify.

13. $\sqrt{225}$
14. $\pm\sqrt{169}$
15. $\sqrt{-(-7)^2}$
16. $\sqrt{(-18)^2}$
17. $\sqrt[3]{-27}$
18. $\sqrt[7]{-128}$
19. $\sqrt{\frac{1}{16}}$
20. $\sqrt[3]{\frac{1}{125}}$
21. $\sqrt{0.25}$
22. $\sqrt[3]{-0.064}$
23. $\sqrt[4]{z^8}$
24. $-\sqrt[6]{x^6}$
25. $\sqrt{49m^6}$
26. $\sqrt{64a^8}$
27. $\sqrt[3]{27r^3}$
28. $\sqrt[3]{-c^6}$
29. $\sqrt{(5g)^4}$
30. $\sqrt[3]{(2z)^6}$
31. $\sqrt{25x^4y^6}$
32. $\sqrt{36x^4z^4}$
33. $\sqrt{169x^8y^4}$
34. $\sqrt{9p^{12}q^6}$
35. $\sqrt[3]{8a^3b^3}$
36. $\sqrt[3]{-27c^9d^{12}}$

Use a calculator to approximate each value to three decimal places.

37. $\sqrt{129}$
38. $-\sqrt{147}$
39. $\sqrt{0.87}$
40. $\sqrt{4.27}$
41. $\sqrt[3]{59}$
42. $\sqrt[3]{-480}$
43. $\sqrt[4]{602}$
44. $\sqrt[5]{891}$
45. $\sqrt[6]{4123}$
46. $\sqrt[7]{46{,}815}$
47. $\sqrt[6]{(723)^3}$
48. $\sqrt[4]{(3500)^2}$

49. **AEROSPACE** The radius r of the orbit of a satellite is given by $r = \sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of the central object, and t is the time it takes the satellite to complete one orbit. Find the radius of the orbit if G is 6.67×10^{-11} N $\cdot$ m^2/kg^2, M is 5.98×10^{24} kg, and t is 2.6×10^6 seconds.

50. **SHOPPING** A certain store found that the number of customers that will attend a limited time sale can be modeled by $N = 125\sqrt[3]{100Pt}$, where N is the number of customers expected, P is the percent of the sale discount, and t is the number of hours the sale will last. Find the number of customers the store should expect for a sale that is 50% off and will last four hours.

EXTRA PRACTICE
See pages 906 and 932.
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H.O.T. Problems

51. **OPEN ENDED** Write a number whose principal square root and cube root are both integers.

52. **REASONING** Determine whether the statement $\sqrt[4]{(-x)^4} = x$ is *sometimes*, *always*, or *never* true.

53. CHALLENGE Under what conditions is $\sqrt{x^2 + y^2} = x + y$ true?

54. REASONING Explain why it is not always necessary to take the absolute value of a result to indicate the principal root.

55. *Writing in Math* Refer to the information on page 402 to explain how nth roots apply to geometry. Analyze what happens to the value of r as the value of V increases.

STANDARDIZED TEST PRACTICE

56. ACT/SAT Which of the following is closest to $\sqrt[3]{7.32}$?

A 1.8

B 1.9

C 2.0

D 2.1

57. REVIEW What is the product of the complex numbers $(5 + i)$ and $(5 - i)$?

F 24

G 26

H $25 - i$

J $26 - 10i$

Spiral Review

Graph each function. State the domain and range. (Lesson 7-3)

58. $y = \sqrt{x - 2}$ **59.** $y = \sqrt{x} - 1$ **60.** $y = 2\sqrt{x} + 1$

61. Determine whether the functions $f(x) = x - 2$ and $g(x) = 2x$ are inverse functions. (Lesson 7-2)

Simplify. (Lesson 5-4)

62. $(3 + 2i) - (1 - 7i)$ **63.** $(8 - i)(4 - 3i)$ **64.** $\dfrac{2 + 3i}{1 + 2i}$

Solve each system of equations. (Lesson 3-2)

65. $2x - y = 7$
$x + 3y = 0$

66. $4x + y = 7$
$3x + \frac{4}{5}y = 5.5$

67. $\frac{1}{4}x + \frac{2}{3}y = 3$
$2x + y = -2$

68. BUSINESS A dry cleaner ordered 7 drums of two different types of cleaning fluid. One type costs \$30 per drum, and the other type costs \$20 per drum. The total cost was \$160. How much of each type of fluid did the company order? Write a system of equations and solve by graphing. (Lesson 3-1)

Graph each function. (Lesson 2-6)

69. $f(x) = 5$ **70.** $f(x) = |x - 3|$ **71.** $f(x) = |2x| + 3$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product. (Lesson 6-2)

72. $(x + 3)(x + 8)$ **73.** $(y - 2)(y + 5)$ **74.** $(a + 2)(a - 9)$

75. $(a + b)(a + 2b)$ **76.** $(x - 3y)(x + 3y)$ **77.** $(2w + z)(3w - 5z)$